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Transient behaviour of multipass shell-and-tube heat exchangers†

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Abstract—A method for predicting the transient responses to arbitrary inlet temperature changes of multipass shell-and-tube heat exchangers with arbitrary number of tubeside passes is developed. The tubeside as well as shellside surface areas and heat transfer coefficients are allowed to be different from pass to pass. The thermal capacities of both fluids and the wall are included. Both possible flow arrangements are considered. The inlet temperature changes may take place on either side or simultaneously on both sides. Generally, the optimum value of M of the summed series terms for the numerical inverse Laplace transform falls in the range $8 \leqslant M \leqslant 20$.

INTRODUCTION

THE TRANSIENT operation of heat exchangers is of increasing interest in industry and research, either for process control applications or for the determination of average heat transfer coefficients in heat exchangers. There exist many references on the dynamic response of shell-and-tube heat exchangers. Most of them, however, focus on the transient behaviour of parallel flow or counterflow heat exchangers. In other words, one can find few research papers which deal with the dynamic process of shell-and-tube heat exchangers with more than one tubeside pass, although such apparatuses are extensively used in industry. Roppo and Ganic [1] as well as Correa and Marchetti [2] applied the cell model to describe dynamic responses to a step inlet change of multipass shell-and-tube heat exchangers. The first paper neglects the influence of the thermal capacity of the core wall and the latter considers this influence, introducing an equivalent tubeside specific heat capacity. The essence of both papers is the application of the finite difference method.

On the basis of the previous work [3], this paper analyses the transient behaviour of shell-and-tube heat exchangers with N tubeside passes (designated as $1-N$) and two different tubeside flow arrangements. It is allowed that the tubeside and shellside heat transfer coefficients as well as surface areas vary from pass to pass and that arbitrary inlet temperature changes (with the exception of discontinuities and rapid oscillations) occur on either side or simultaneously on both sides.

† Dedicated to Professor Dr.-Ing. Dr.-Ing.e.h. Ulrich Grigull.

DEVELOPMENT OF THE GOVERNING EQUATIONS

The shell-and-tube heat exchanger under consideration is illustrated schematically in Fig. 1 where the number N of tubeside passes is arbitrary (even or odd) and two possible tubeside flow arrangements are presented. To simplify the derivation of the governing equations, the following assumptions are necessary:

1. The thermal flow rates $\dot{W}_1$ and $\dot{W}_2$ of the fluids are constant throughout and the heat transfer coefficient is constant within any tubeside pass, but it may vary with the tubeside pass.
2. The shellside fluid is completely mixed at any cross-section of its nominal flow path and no bypassing occurs.
3. All thermal properties are constant.
4. Longitudinal heat conduction within the wall is neglected and the wall heat transfer resistance is negligible compared with convective heat transfer resistances.
5. No heat is transferred from the shell of the exchanger to the environment and there exists no influence of the thermal capacity of the shell on the transient process.

To develop a general coordinate system, the origin of the coordinate is always set at the location where the shellside fluid enters the heat exchanger. According to the above idealizations, one can derive the $(2N+1)$ partial differential equations which describe the transient behaviour of $1-N$ shell-and-tube heat exchangers

$$\dot{W}_1 \frac{\partial t_1}{\partial x} + C_1 \frac{\partial t_1}{\partial \tau} + \sum_{i=1}^{N} (hA)_{1i}(t_1 - t_{wi}) = 0 \qquad (1)$$

NOMENCLATURE

- A heat transfer surface area [m^2]
- C heat capacity [J K^{-1}]
- $f_1(z), f_2(z)$ inlet temperature changes
- $F_1(s), F_2(s)$ transformed forms of $f_1(z)$ and $f_2(z)$ in the image domain
- h heat transfer coefficient [W m^{-2} K^{-1}]
- l distance from the entrance of shellside fluid [m]
- L length of the heat exchanger [m]
- M number of summed series terms in equation (21)
- N number of tubeside passes
- NTU number of transfer units [dimensionless]
- s parameter of the Laplace transform
- t dimensionless temperature, $t = (\theta - \theta_0)/(\theta_r - \theta_0)$
- T transformed form of t in the Laplace transform domain
- $\dot{W}$ thermal flow rate [W K^{-1}]
- x dimensionless coordinate, $x = l/L$
- z dimensionless time

Greek symbols

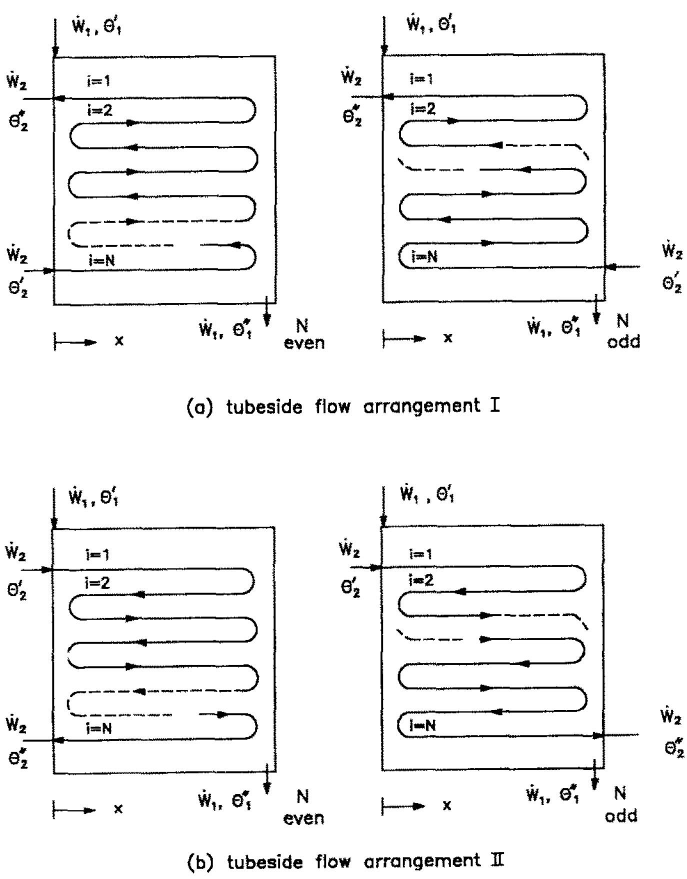
- α parameter in equation (11)
- θ temperature [K]
- θ_0 initial temperature in heat exchangers [K]
- θ_r reference temperature [K]
- λ eigenvalues
- τ time [s]
- τ_r residence time of fluid in the heat exchanger [s].

Subscripts

- 1 shellside fluid
- 2 tubeside fluid
- e exit
- w core wall of the heat exchanger.

Superscripts

- $'$ inlet
- $''$ exit.



(a) tubeside flow arrangement I

(b) tubeside flow arrangement II

FIG. 1. Schematic representation of multipass shell-and-tube heat exchangers.

$$\pm(-1)^i \dot{W}_{2i}\frac{\partial t_{2i}}{\partial x}+C_{2i}\frac{\partial t_{2i}}{\partial \tau}+(hA)_{2i}(t_{2i}-t_{wi})=0 \quad (i=1,2,\ldots,N) \quad (2)$$

$$C_{wi}\frac{\partial t_{wi}}{\partial \tau}-(hA)_{1i}(t_1-t_{wi})-(hA)_{2i}(t_{2i}-t_{wi})=0 \quad (i=1,2,\ldots,N) \quad (3)$$

where the positive sign (+) and negative sign (−) of (±) in equation (2) are valid for the tubeside flow arrangements I and II which are shown in Fig. 1, respectively. For an incompressible fluid, it must be satisfied that $\dot{W}_{2i}=\dot{W}_2$, i.e., the tubeside thermal flow rate does not vary with the pass. However, the tubeside thermal capacity C_{2i} may be different from pass to pass. The residence times τ_{r1} and τ_{r2} of both fluids and some dimensionless parameters are introduced

$$\tau_{r1}=\frac{C_1}{\dot{W}_1},\quad \tau_{r2}=\frac{C_2}{\dot{W}_2},\quad \tau_{r2i}=\frac{C_{2i}}{\dot{W}_{2i}},$$

$$R_1=\frac{\dot{W}_1}{\dot{W}_2}=\frac{1}{R_2},\quad U_1=\frac{(hA)_1}{\dot{W}_1},\quad U_2=\frac{(hA)_2}{\dot{W}_2},$$

$$U_{1i}=\frac{(hA)_{1i}}{\dot{W}_1},\quad U_{2i}=\frac{(hA)_{2i}}{\dot{W}_2}$$

$$NTU_1=\left[\frac{1}{(hA)_1}+\frac{1}{(hA)_2}\right]^{-1}\frac{1}{\dot{W}_1}=\frac{U_1U_2R_2}{U_1+U_2R_2}$$

$$\varepsilon_{1i}=\frac{(hA)_{1i}}{(hA)_1}=\frac{U_{1i}}{U_1},\quad \varepsilon_{2i}=\frac{(hA)_{2i}}{(hA)_2}=\frac{U_{2i}}{U_2},$$

$$\varepsilon_{ci}=\frac{C_{2i}}{C_2},\quad \varepsilon_{wi}=\frac{C_{wi}}{C_w}$$

where

$$(hA)_1=\sum_{i=1}^{N}(hA)_{1i},\quad (hA)_2=\sum_{i=1}^{N}(hA)_{2i},$$

$$C_2=\sum_{i=1}^{N}C_{2i}\quad \text{and}\quad C_w=\sum_{i=1}^{N}C_{wi}.$$

Obviously, we have the following relationships:

$$\sum_{i=1}^{N}\varepsilon_{1i}=1,\quad \sum_{i=1}^{N}\varepsilon_{2i}=1,\quad \sum_{i=1}^{N}\varepsilon_{ci}=1,$$

$$\sum_{i=1}^{N}\varepsilon_{wi}=1\quad \text{and}\quad \tau_{r2i}=\varepsilon_{ci}\tau_{r2}. \quad (4)$$

Other dimensionless parameters are defined as

$$R_\tau=\frac{\tau_{r2}}{\tau_{r1}},\quad R_{c1}=\frac{C_1}{C_2}=\frac{R_1}{R_\tau}=\frac{1}{R_{c2}}$$

$$R_w=\frac{C_w}{C_1+C_2},\quad R_{wi}=\frac{C_{wi}}{C_1+C_2}=\varepsilon_{wi}R_w$$

$$\alpha_{1i}=\frac{U_{1i}}{1+R_{c2}},\quad \alpha_{2i}=\frac{U_{2i}}{R_\tau(1+R_{c1})},$$

$$\text{sign}=\pm(-1)^i.$$

The dimensionless time variable z is introduced as

$$z=\frac{\tau}{\tau_{r1}}. \quad (5)$$

By means of the above dimensionless variable and parameters, equations (1)–(3) can be rewritten as follows:

$$\frac{\partial t_1}{\partial x}+\frac{\partial t_1}{\partial z}+\sum_{i=1}^{N}U_{1i}(t_1-t_{wi})=0 \quad (6)$$

$$\text{sign}\frac{\partial t_{2i}}{\partial x}+\varepsilon_{ci}R_\tau\frac{\partial t_{2i}}{\partial z}+U_{2i}(t_{2i}-t_{wi})=0 \quad (i=1,2,\ldots,N) \quad (7)$$

$$R_{wi}\frac{\partial t_{wi}}{\partial z}-\alpha_{1i}(t_1-t_{wi})-\alpha_{2i}(t_{2i}-t_{wi})=0 \quad (i=1,2,\ldots,N). \quad (8)$$

These dimensionless partial differential equations are subject to the following initial conditions:

$$t_1(x,0)=0,\quad t_{2i}(x,0)=t_{wi}(x,0)=0 \quad (i=1,2,\ldots,N). \quad (9)$$

The shellside arbitrary inlet temperature change can be described as

$$t_1(0,z)=f_1(z) \quad (10)$$

and the other N boundary and interface conditions pertinent to the tubeside flow are listed in Table 1. They vary with the number of tubeside passes and flow arrangements.

Functions $f_1(z)$ and $f_2(z)$ describe any possible inlet temperature changes on both sides of multipass shell-and-tube heat exchangers, which may occur separately or simultaneously. The most common forms of such changes may assume step, ramp, exponential or periodic expressions

Table 1. The boundary and interface conditions for $t_{2i}(x,z)$

	$x=0,\quad z\geqslant 0$	$x=1,\quad z\geqslant 0$	Tubeside flow arrangement I	II
N even	$t_{2i}=t_{2i+1}=t_{2i,i+1}$ $i=2,4,\ldots,N-2$	$t_{2i}=t_{2i+1}=t_{2i,i+1}$ $i=1,3,\ldots,N-1$	$t_{2N}(0,z)=f_2(z)$	$t_{21}(0,z)=f_2(z)$
N odd	$t_{2i}=t_{2i+1}=t_{2i,i+1}$ $i=2,4,\ldots,N-1$	$t_{2i}=t_{2i+1}=t_{2i,i+1}$ $i=1,3,\ldots,N-2$	$t_{2N}(1,z)=f_2(z)$	$t_{21}(0,z)=f_2(z)$

$$f_1(z)\ \text{or}\ f_2(z) = \begin{cases} 1 & \text{step} \\ \alpha z & \text{ramp} \\ \exp(-\alpha z) & \text{exponential} \\ \sin(\alpha z) & \text{periodic} \end{cases} \tag{11}$$

where α is a known parameter.

TRANSIENT RESPONSES

To obtain the transient responses to the given inlet temperature changes, the solution to equations (6)–(8) must be derived. For this purpose, one can take advantage of the Laplace transform of these equations using s as the Laplace parameter with respect to z. According to the given initial conditions, the transformed equations are as follows:

$$\frac{dT_1}{dx} = -\left(S+U_1-\sum_{i=1}^{N}\frac{U_1\varepsilon_{1i}\alpha_{1i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}}\right)T_1 + \sum_{i=1}^{N}\frac{U_1\varepsilon_{2i}\alpha_{2i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}}T_{2i} \tag{12}$$

$$\frac{dT_{2i}}{dx} = \operatorname{sign}\frac{U_2\varepsilon_{2i}\alpha_{1i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}}T_1 + \operatorname{sign}\left(\frac{U_2\varepsilon_{2i}\alpha_{2i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}}-\varepsilon_{ci}R_\tau s-U_2\varepsilon_{2i}\right)T_{2i} \quad (i=1,2,\ldots,N) \tag{13}$$

$$T_{wi} = \frac{\alpha_{1i}T_1+\alpha_{2i}T_{2i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}} \quad (i=1,2,\ldots,N). \tag{14}$$

With the corresponding interface and boundary conditions, equations (12) and (13) compose a closed system consisting of $(N+1)$ homogeneous ordinary differential equations of the first order. In matrix notation, this homogeneous system can be expressed in the form

$$\frac{d\mathbf{T}}{dx} = \mathbf{AT} \tag{15}$$

where $\mathbf{T} = (T_{21}, T_{22}, \ldots, T_{2N}, T_1)^T$ and $\mathbf{A}$ is an $(N+1)\times(N+1)$ matrix, the elements of which are as follows:

$$a_{ij} = \begin{cases} 0 & i \neq j \\ \operatorname{sign}\left(\dfrac{U_2\varepsilon_{2i}\alpha_{2i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}} \right. & i,j \leqslant N \\ \left. -\varepsilon_{ci}R_\tau s-U_2\varepsilon_{2i}\right) & i=j \end{cases}$$

$$a_{i,N+1} = \operatorname{sign}\frac{U_2\varepsilon_{2i}\alpha_{1i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}}, \quad i=1,2,\ldots,N$$

$$a_{N+1,j} = \frac{U_2\varepsilon_{2i}\alpha_{2i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}}, \quad j=1,2,\ldots,N$$

$$a_{N+1,N+1} = -(s+U_1)+\sum_{i=1}^{N}\frac{U_1\varepsilon_{1i}\alpha_{1i}}{R_{wi}s+\alpha_{1i}+\alpha_{2i}}.$$

In the light of the similar procedure described in ref. [4], a general solution to the system (15) is derived as

$$\mathbf{T} = \sum_{j=1}^{N+1} d_j B_j \exp(\lambda_j x) \tag{16}$$

where λ_j and B_j $(j = 1,2,\ldots,N+1)$ are distinct eigenvalues and the corresponding eigenvectors, respectively, $B_j = (b_{1j}, b_{2j}, \ldots, b_{N+1,j})^T$. Generally, the eigenvalues are different from each other. If there exist multiple eigenvalues, the solution (16) may fail. In this case one should refer to the literature [4]. To obtain the particular solution subject to the given interface and boundary conditions, $(N+1)$ unknown coefficients d_j $(j = 1,2,\ldots,N+1)$ must be determined. From equation (16) and the given conditions, one can find a matrix equation which confines these coefficients

$$\mathbf{WD} = \mathbf{G} \tag{17}$$

where $\mathbf{D} = (d_1, d_2, \ldots, d_{N+1})^T$ and $\mathbf{G} = (0, 0, \ldots, 0, F_2(s), F_1(s))^T$, if the inlet boundary conditions on both sides are taken as the last two equations. $\mathbf{W}$ is an $(N+1)\times(N+1)$ matrix whose elements depend on many factors such as the number of tubeside passes, the tubeside flow arrangement and the multiplicity of eigenvalues λ_j. Consequently, the coefficient vector $\mathbf{D}$ is obtained

$$\mathbf{D} = \mathbf{W}^{-1}\mathbf{G}. \tag{18}$$

On determining the coefficients d_j in equation (16), one has found the particular solution subject to the given conditions in the image domain of the Laplace transform. In this domain, the exit transient response T_{1e} of the shellside flow is drawn from equation (16)

$$T_{1e} = \sum_{j=1}^{N+1} d_j b_{N+1,j} \exp(\lambda_j) \tag{19}$$

and the tubeside exit response T_{2e} varies with the tubeside flow arrangement and the number N of tubeside passes. For the tubeside flow arrangement I

$$T_{2e} = T_{21} = \sum_{j=1}^{N+1} d_j b_{1j}. \tag{20a}$$

For the tubeside flow arrangement II

$$T_{2e} = T_{2N}\begin{cases} \sum\limits_{j=1}^{N+1} d_j b_{Nj} & \text{even } N \\ \sum\limits_{j=1}^{N+1} d_j b_{Nj}\exp(\lambda_j) & \text{odd } N. \end{cases} \tag{20b}$$

Obviously, d_j, B_j and λ_j may all be functions of the Laplace parameter s. It is impossible to perform the inverse transform of the above mentioned expressions analytically. A numerical inverse Laplace transform is used to derive the transient responses to the inlet temperature changes in the time domain. This numerical inversion method, called the Gaver–Stehfest algorithm [5], can be described by the following expressions:

$$f(z) = \frac{\ln 2}{z} \sum_{i=1}^{M} K_i F\left(\frac{\ln 2}{z} i\right)$$

$$K_i = (-1)^{i+M/2} \sum_{k=(i+1)/2}^{\min(i, M/2)} \times \frac{k^{M/2}(2k)!}{(M/2-k)!\,k!\,(k-1)!\,(i-k)!\,(2k-i)!} \quad (21)$$

where M must be even. The word 'min' means that the number of summed series terms takes the lower of i and $M/2$.

By means of the expression (21), one can obtain the transient temperature profiles of both fluids and of the core wall as well as the exit responses to a given inlet temperature change according to equations (16), (19) and (20).

EXAMPLES AND DISCUSSION

In order to examine the method presented in this paper, the transient responses to the inlet temperature changes described in equation (11) of the heat exchanger with parallel and countercurrent flow were computed and the same results as in ref. [3] were obtained. Under the following parameters that $R_w = 1.0$, $R_\tau = 1.0$, $R_1 = 1.0$, $U_1 = U_2 = 2NTU_1$ and $\alpha = 1.0$, other examples of the exit transient behaviours have

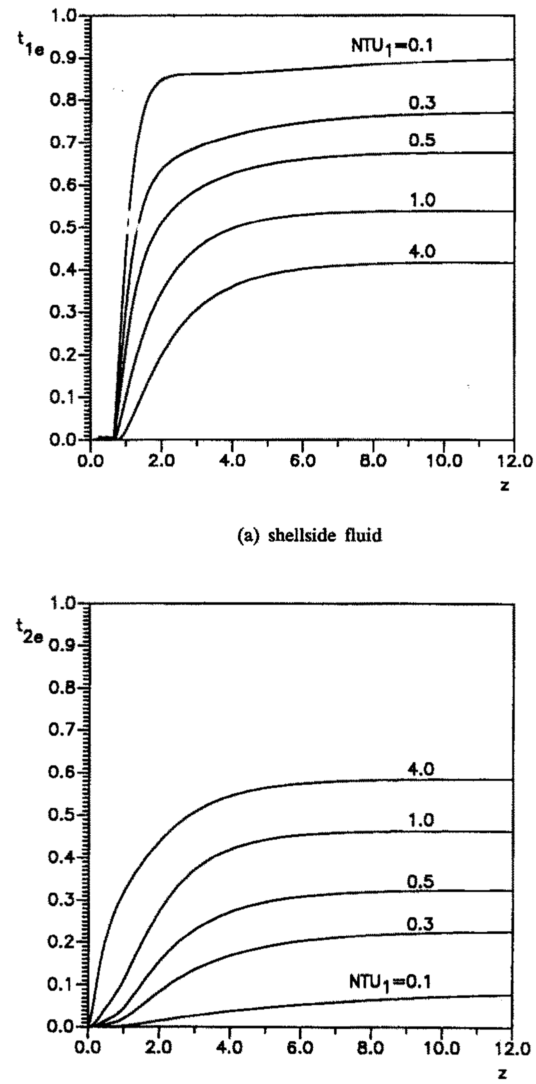


FIG. 2. Exit responses to a shellside step inlet temperature change of 1–2 heat exchanger with tubeside flow arrangement I ($\varepsilon_{11} = \varepsilon_{12} = 0.5$, $\varepsilon_{21} = \varepsilon_{22} = 0.5$, $\varepsilon_{c1} = \varepsilon_{c2} = 0.5$, $\varepsilon_{w1} = \varepsilon_{w2} = 0.5$). (a) Shellside fluid. (b) Tubeside fluid.

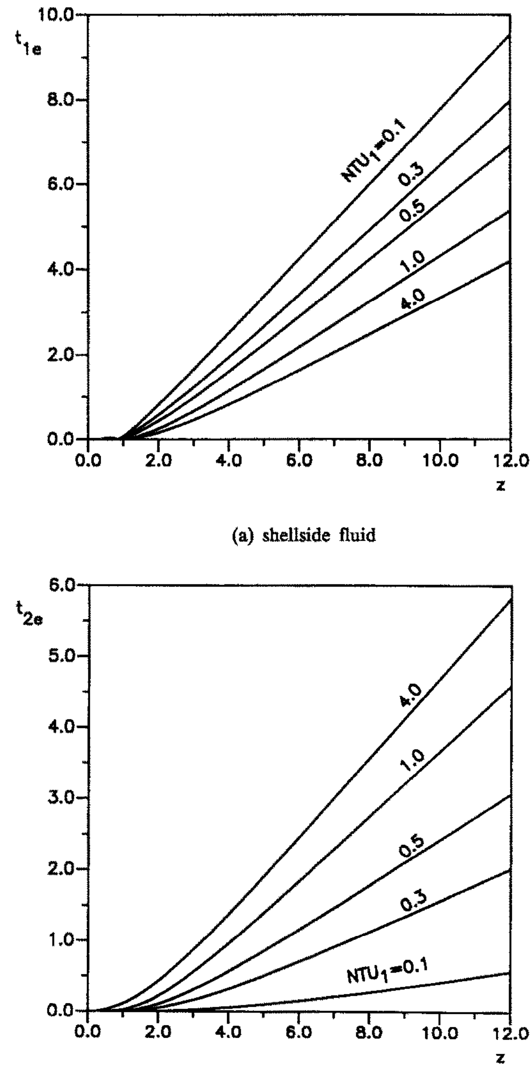
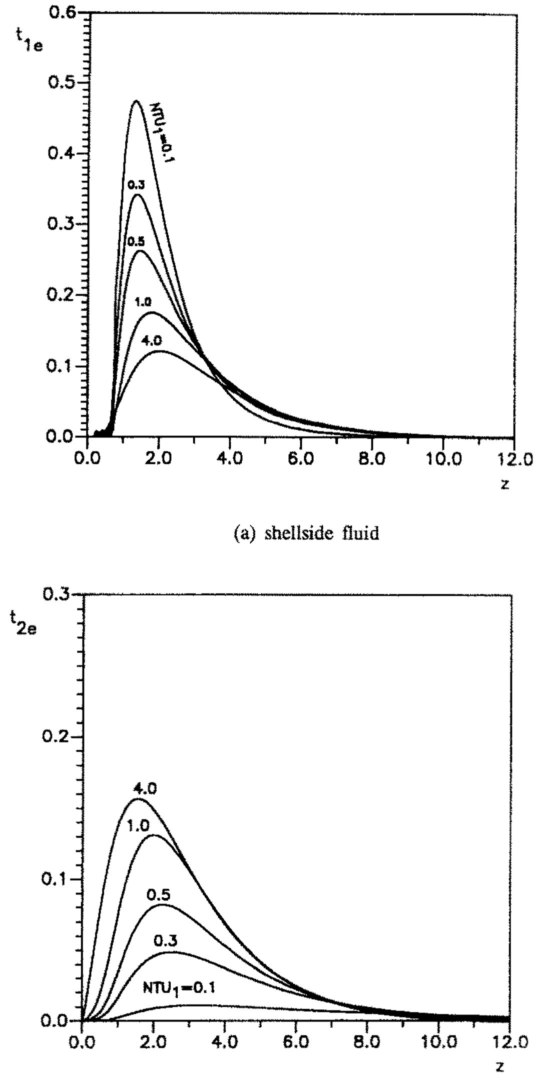


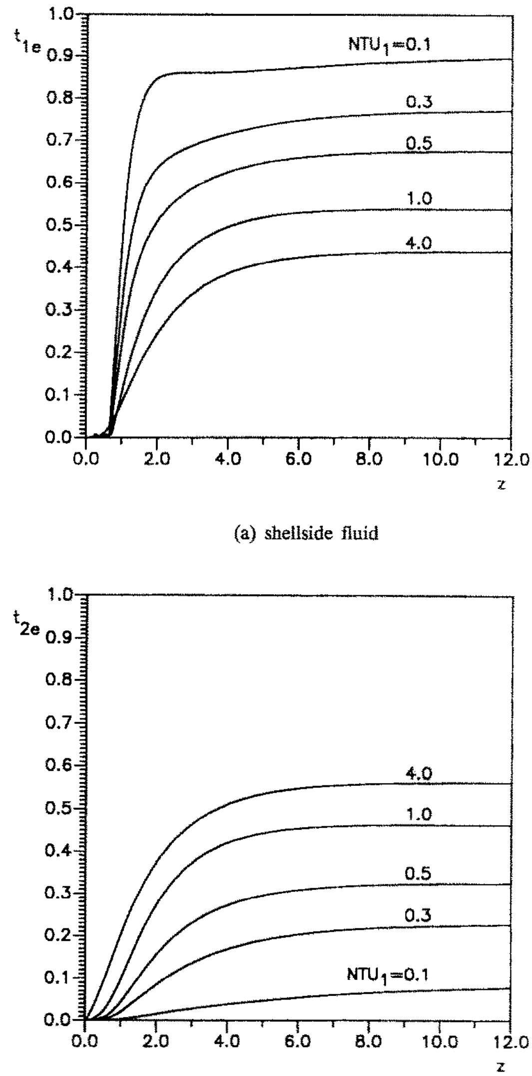
FIG. 3. Exit responses to a shellside ramp inlet temperature change of 1–4 heat exchanger with tubeside flow arrangement I ($\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{13} = \varepsilon_{14} = 0.25$, $\varepsilon_{21} = \varepsilon_{22} = \varepsilon_{23} = \varepsilon_{24} = 0.25$, $\varepsilon_{c1} = \varepsilon_{c2} = \varepsilon_{c3} = \varepsilon_{c4} = 0.25$, $\varepsilon_{w1} = \varepsilon_{w2} = \varepsilon_{w3} = \varepsilon_{w4} = 0.25$). (a) Shellside fluid. (b) Tubeside fluid.



(a) shellside fluid

(b) tubeside fluid

FIG. 4. Exit responses to a shellside exponential inlet temperature change of 1–6 heat exchanger with tubeside flow arrangement II ($\varepsilon_{11} = \varepsilon_{14} = 0.2$, $\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{15} = \varepsilon_{16} = 0.15$, $\varepsilon_{21} = \varepsilon_{24} = 0.2$, $\varepsilon_{22} = \varepsilon_{23} = \varepsilon_{25} = \varepsilon_{26} = 0.15$, $\varepsilon_{c1} = \varepsilon_{c4} = 0.2$, $\varepsilon_{c2} = \varepsilon_{c3} = \varepsilon_{c5} = \varepsilon_{c6} = 0.15$, $\varepsilon_{w1} = \varepsilon_{w4} = 0.2$, $\varepsilon_{w2} = \varepsilon_{w3} = \varepsilon_{w5} = \varepsilon_{w6} = 0.15$). (a) Shellside fluid. (b) Tubeside fluid.



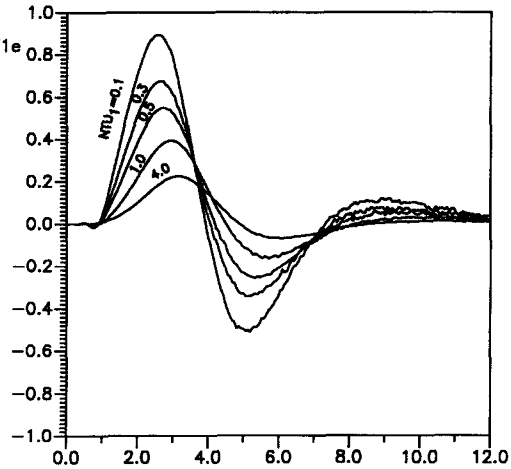
(a) shellside fluid

(b) tubeside fluid

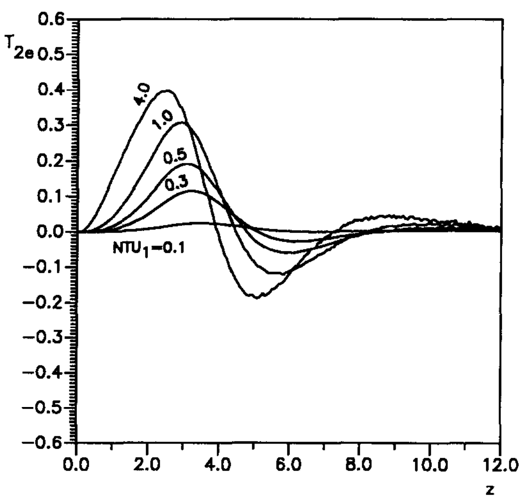
FIG. 5. Exit responses to a shellside step inlet temperature change of 1–8 heat exchanger with tubeside flow arrangement I ($\varepsilon_{11} = \varepsilon_{12} = \cdots = \varepsilon_{18} = 0.125$, $\varepsilon_{21} = \varepsilon_{22} = \cdots = \varepsilon_{28} = 0.125$, $\varepsilon_{c1} = \varepsilon_{c2} = \cdots = \varepsilon_{c8} = 0.125$, $\varepsilon_{w1} = \varepsilon_{w2} = \cdots = \varepsilon_{w8} = 0.125$). (a) Shellside fluid. (b) Tubeside fluid.

been calculated for shell-and-tube heat exchangers with more tubeside passes, both different tubeside flow arrangements and different distribution ratios of $(hA)_1$ as well as $(hA)_2$. The results are illustrated in Figs. 2–6. Meanwhile, the results have shown that the energy balance between both fluids is precisely satisfied in the steady-state for the step inlet temperature change, which demonstrates that the algorithm for numerical inverse Laplace transform can be applied for predicting the transient behaviour of multipass heat exchangers. The comparison for such cases is not possible because no data are available in the literature.

The computation has shown that for both tubeside flow arrangemens, almost the same exit responses appear if the number N of tubeside passes is greater than or equal to 6 ($N \geqslant 6$). This phenomenon is similar to the relationship between multipass shell-and-tube and transversely mixed cross-flow heat exchangers in the stationary state for a higher number of tubeside passes. As shown in these figures, no immediate exit behaviours respond to any sudden changes taking place at the inlets and there exist time lags between the inlet and exit changes to some extent because of the influence of thermal capacities of both fluids and the core wall. This kind of lag lasts some time within $z \leqslant 1.0$ if the inlet temperature change occurs only on the shellside. Within this time lag, there



(a) shellside fluid



(b) tubeside fluid

FIG. 6. Exit responses to a shellside sinusoidal inlet temperature change of 1–3 heat exchanger with tubeside flow arrangement I ($\varepsilon_{11} = \varepsilon_{13} = 0.4$, $\varepsilon_{12} = 0.2$, $\varepsilon_{21} = \varepsilon_{23} = 0.4$, $\varepsilon_{22} = 0.2$, $\varepsilon_{c1} = 0.38$, $\varepsilon_{c2} = 0.25$, $\varepsilon_{c3} = 0.37$, $\varepsilon_{w1} = 0.35$, $\varepsilon_{w2} = 0.3$, $\varepsilon_{w3} = 0.35$). (a) Shellside fluid. (b) Tubeside fluid.

may appear minor vibrations of the computed results of the exit temperature responses and one needs to pay attention to these results. The reason is that greater round-off errors may arise within the time lag. Since there should exist no exit responses within this time lag, in fact, it is easy to distinguish and eliminate these numerical vibrations.

As pointed out in ref. [3], one should not expect to find an optimum value of M of the summed series terms which is suitable for all cases. The optimum value of M may vary with the inlet temperature changes, the number N of tubeside passes, flow arrangements and some other factors. A method how to determine an appropriate value of M has been discussed [3]. Generally, the optimum value of M for the inverse transform lies in the range $8 \leqslant M \leqslant 20$. The inlet temperature changes can take arbitrary forms except for the prerequisites that they have no discontinuities or rapid oscillations.

As shown in Fig. 6 and pointed out by Jacquot *et al.* [6], one may not find the accurate responses to a high frequency sinusoidal inlet temperature change for a greater value of the dimensionless time. In fact, the calculated exit response to such a sinusoidal change approaches zero by means of the Gaver–Stehfest algorithm if $z > 100$. Obviously, this is physically false. This means that the above-mentioned algorithm does not work well for inlet temperature changes which have rapid oscillatory components.

CONCLUSIONS

The method of numerical inverse Laplace transform has been successfully applied to describe the transient behaviour of multipass shell-and-tube heat exchangers with different tubeside flow arrangements and changeable distribution ratios of $(hA)_1$ as well as $(hA)_2$. Arbitrary inlet temperature changes are allowed to take place on either side or simultaneously on both sides.

Owing to the influence of thermal capacities of fluids and the core wall, there appears to be a time lag beween the inlet temperature changes and exit responses. The span of this time lag depends on other factors such as the tubeside flow arrangement and NTU values, besides the thermal capacities of fluids and the core wall. Within this lag, there may occur numerical vibrations which can easily be distinguished and eliminated according to the characteristics of the transient behaviours of apparatuses. One has to select an appropriate value of M of the summed series terms for the numerical inverse transform. The optimum M lies in the range $8 \leqslant M \leqslant 20$. One should be careful when applying the Gaver–Stehfest algorithm to calculating the exit responses to oscillatory inlet changes.

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COMPORTEMENT VARIABLE DES ECHANGEURS THERMIQUES MULTIPASSES TUBE-CALANDRE

Résumé—On développe une méthode pour prédire les réponses variables à des changements arbitraires de température à l'entrée des échangeurs thermiques multipasses tube-calandre avec un nombre arbitraire de passes côté tube. Les aires des surfaces côté tube et côté calandre ainsi que les coefficients de transfert thermique sont supposés différents d'une passe à l'autre. Les capacités thermiques des fluides et de la paroi sont incluses. On considère les arrangements possibles d'écoulement. Les changements de température d'entrée peuvent prendre place sur chaque côté ou simultanément sur les deux côtés. Généralement, la valeur optimale M de la somme des termes de la série pour la transformée inverse numérique de Laplace tombe dans le domaine $8 \leqslant M \leqslant 20$.

INSTATIONÄRES VERHALTEN VON MEHRGÄNGIGEN ROHRBÜNDELWÄRMEÜBERTRAGERN

Zusammenfassung—Es wird eine Methode zur Berechnung des instationären Verhaltens von mehrgängigen Rohrbündelwärmeübertragern bei beliebiger Änderung der Eintrittstemperaturen entwickelt. Die Anzahl der rohrseitigen Durchgänge darf beliebig sein. Die Flächen und sowohl die inneren als auch die äußeren Wärmeübergangskoeffizienten der einzelnen Durchgänge dürfen sich unterscheiden. Die Wärmekapazitäten von beiden Fluiden und der Wand werden berücksichtigt. Beide möglichen Schaltungsarten werden betrachtet. Es dürfen sich beide Eintritts-temperaturen einzeln oder gleichzeitig ändern. Die Zahl M der Summanden bei der numerischen Laplace-Rücktransformation liegt im Bereich $8 \leqslant M \leqslant 20$.

НЕСТАЦИОНАРНЫЕ ХАРАКТЕРИСТИКИ МНОГОХОДОВЫХ КОЖУХОТРУБНЫХ ТЕПЛООБМЕННИКОВ

Аннотация—Разработан метод определения нестационарного отклика на произвольные изменения температуры на входе многоходовых кожухотрубных теплообменников с произвольным количеством ходов труб. Площади поверхностей труб и кожухов, а также коэффициенты теплопередачи изменяются от хода к юоду. Учитываются теплоемкости жидкости и стенки. Исследуются две возможные структуры течения, соответствующие случаям, когда температура на входе может изменяться как с одной стороны, так и одновременно с обеих. Как правило, оптимальное значение M суммы членов ряда при численном обратном преобразовании Лапласа изменяется в диапазоне $8 \leqslant M \leqslant 20$.